

# Unitary limit and quantum interference effect in disordered two-dimensional crystals with nearly half-filled bands

Y. H. Yang,<sup>1,2</sup> D. Y. Xing,<sup>2</sup> Y. G. Wang,<sup>1</sup> and M. Liu,<sup>1</sup>

<sup>1</sup>*Department of Physics, Southeast University, Nanjing 210096, China*

<sup>2</sup>*National Laboratory of Solid State Microstructures, Nanjing University, Nanjing 210008, China*

## Abstract

Based on the self-consistent  $T$ -matrix approximation, the quantum interference (QI) effect is studied with the diagrammatic technique in weakly-disordered two-dimensional crystals with nearly half-filled bands. In addition to the usual 0-mode cooperon and diffuson, there exist  $\pi$ -mode cooperon and diffuson in the unitary limit due to the particle-hole symmetry. The diffusive  $\pi$ -modes are gapped by the deviation from the exactly-nested Fermi surface. The conductivity diagrams with the gapped  $\pi$ -mode cooperon or diffuson are found to give rise to unconventional features of the QI effect. Besides the inelastic scattering, the thermal fluctuation is shown to be also an important dephasing mechanism in the QI processes related with the diffusive  $\pi$ -modes. In the proximity of the nesting case, a power-law anti-localization effect appears due to the  $\pi$ -mode diffuson. For large deviation from the nested Fermi surface, this anti-localization effect is suppressed, and the conductivity remains to have the usual logarithmic weak-localization correction contributed by the 0-mode cooperon. As a result, the dc conductivity in the unitary limit becomes a non-monotonic function of the temperature or the sample size, which is quite different from the prediction of the usual weak-localization theory.

PACS number(s): 73.20.Fz, 71.30.+h, 72.15.Rn, 73.20.Jc

# I. Introduction

Since the pioneering work of Anderson [1], the disorder-induced localization in electronic systems has been the subject of intense research activities for more than four decades. According to the scaling theory proposed by Abrahams et al. [2], all electronic states in one- and two-dimensional (1D and 2D) disordered systems are localized irrespective of the degree of the randomness, and in 3D systems there exist the metal-insulator transitions. The prediction of the scaling theory for 1D systems is in agreement with the exact results [3, 4], and the existence of Anderson localization in 3D systems is widely accepted [5]. The dimension 2 is the marginal dimension in the problem of Anderson localization. It is believed that in generic situations, there exist no true metallic states in 2D disordered noninteracting systems at zero temperature. However, some exceptions have been known to this rule. For example, non-localized states were found at the band center of a 2D Anderson model with purely off-diagonal disorder [6-10]. The metal-insulator transition is also recently shown to occur in a disordered 2D square lattice model with long-range power-law transfer terms [11]. Therefore, the disorder effects in 2D noninteracting electron systems deserve further investigation.

In the scaling theory of localization, the perturbative approach is exploited to study the weak-disorder limit, revealing the weak-localization effect in 2D electron systems [5, 12, 13]. The weak-localization theory describes the quantum interference (QI) effect that results from constructive interference between the closed electron paths and their time-reversal counterparts, which can be represented by the maximally-crossed diagrams in the diagrammatic approach. While the QI effect contributes no singular correction to the electronic density of states (DOS), the QI correction to the dc conductivity of a 2D system is shown to be [5, 12, 13]

$$\delta\sigma = -\frac{e^2}{2\pi^2} \times \begin{cases} \ln(\gamma_0/\gamma_i), & \text{if } \gamma_i \gg \gamma_L, \\ \ln(\gamma_0/\gamma_L), & \text{if } \gamma_L \gg \gamma_i, \end{cases} \quad (1.1)$$

where  $\gamma_0$  and  $\gamma_i$  denote, respectively, the elastic and inelastic scattering rates, and  $\gamma_L = D/2L^2$  with  $D$  the diffusion coefficient and  $L$  the sample size. At finite temperatures, the inelastic scattering has a dephasing effect on the QI process. Equation (1.1) indicates that the dc conductivity is a monotonic function of the temperature or the sample size, as the inelastic scattering rate  $\gamma_i$  decreases with decreasing the temperature.

Equation (1.1) is usually obtained by means of the free-electron model within the Born approximation, in which only the twofold scattering process from the same impurity is concerned. However, for a 2D crystal with half-filled band, there usually exist the particle-hole symmetry of electronic spectrum, as well as the van Hove singularity in the DOS at the Fermi level. The particle-hole symmetry has been recently shown to play an important role on the QI effect of the quasiparticles in 2D disordered *d*-wave superconductors [14, 15], as well as on the disorder effects in 2D Mott-Hubbard insulators [16]. The large value of the DOS at the Fermi level can drive a disordered system into the unitary limit [17]. Therefore, it is highly desirable to investigate the effects of the particle-hole symmetry and van Hove singularity on the QI process in disordered 2D crystals. While both the Born and unitary limits have been widely investigated in the disordered *d*-wave superconductors, the QI effect in the unitary limit did not receive enough attention in the study of disordered normal metals.

In the previous short report [18], we have studied the QI effect at zero temperature in disordered 2D crystals with exactly half-filled bands. We pointed out that it is necessary to

treat the impurity scattering within the self-consistent  $T$ -matrix approximation (SCTMA) due to the existence of the van Hove singularity at the Fermi level. The SCTMA approach takes into account the effect of multiple-scattering events from the same impurity, and thus gives naturally the results of the Born and the unitary approximations in two distinct limits. In addition to the usual 0-mode cooperon and diffuson, there exist the  $\pi$ -mode cooperon and diffuson in the unitary limit due to the particle-hole symmetry. These diffusive  $\pi$ -modes were found to introduce some new conductivity diagrams, and the resulting QI correction to the conductivity in the unitary limit was shown to increase with the square of the sample size, suggesting the existence of the extended states at the band center.

The aim of this work is to study further the situation of deviation from exact half-filling, and investigate the dephasing effect on the QI process at finite temperatures. Much details of the derivations omitted in Ref. [18] is also presented in this paper. Contrary to the 0-mode cooperon and diffuson, the diffusive  $\pi$ -modes are found to be gapped by the deviation from the exactly-nested Fermi surface. The conductivity diagrams with these gapped diffusive  $\pi$ -modes give rise to unexpected features of the QI effect. In addition to the inelastic scattering, the thermal fluctuation is shown to be also an important dephasing mechanism in the QI processes related with the diffusive  $\pi$ -modes. Upon approaching to the nested Fermi surface, the conductivity is subject to a power-law anti-localization correction induced by the  $\pi$ -mode diffuson. For large deviation from the nesting case, the contributions of the diffusive  $\pi$ -modes are suppressed, and the usual logarithmic weak-localization effect remains due to the 0-mode cooperon. Consequently, the QI correction to the conductivity in the unitary limit has a non-monotonic behavior with increasing the temperature or the sample size. This result is strikingly different from the prediction of the usual weak-localization theory.

The structure of this paper is as follows. In Sec. II, the SCTMA scenario is described in details for a disordered 2D crystal with nearly half-filled band, and the conditions for the Born and unitary limits are presented. Based on the SCTMA, the expressions of the 0-mode and  $\pi$ -mode cooperons and diffusons are derived in Sec. III for the nearly-nested Fermi surface. In Sec. IV, we evaluate the electric current-current correlation function responsible for the QI effect, and then calculate the QI correction to dc conductivity in the unitary limit. The conclusions are summarized in Sec. V. In Appendix A, we prove some mathematical formulas, which are useful in the evaluations of the previous sections. Appendix B is presented to show that the electric current-current correlation function in the retarded-retarded (RR) channel has a vanishing QI correction to the dc conductivity.

## II. Self-consistent $T$ -matrix approximation

We start with the tight-binding model for a square lattice, of which the electrical spectrum is described by

$$\epsilon_{\mathbf{k}} = -2t(\cos k_x a + \cos k_y a) - \mu, \quad (2.1)$$

where  $a$  is the lattice constant,  $t$  is the nearest-neighbor hopping integral, and  $\mu$  is the chemical potential. A half-filled band ( $\mu = 0$ ) is characteristic of a perfectly-nested Fermi surface, which is composed of four straight lines ( $k_x \pm k_y = \pi/a$  and  $k_x \pm k_y = -\pi/a$ ) in the momentum space. The electronic velocity  $\mathbf{v}_{\mathbf{k}} = \nabla \epsilon_{\mathbf{k}}$  is given by

$$v_{\mathbf{k}}^{\alpha} = 2ta \sin k_{\alpha} a, \quad (\alpha = x, y), \quad (2.2)$$

here we take  $\hbar = k_B = 1$ . The DOS per spin of the clean crystal, defined by  $\rho_{\text{cl}}(\epsilon) = \sum_{\mathbf{k}} \delta(\epsilon - \epsilon_{\mathbf{k}})$ , is shown to be of a logarithmic van Hove singularity at the center of the band, i.e.,

$$\rho_{\text{cl}}(\epsilon) = \frac{1}{2\pi^2 a^2 t} \ln \frac{16t}{|\epsilon + \mu|}, \quad \left(\text{for } |\epsilon + \mu| \ll 4t\right). \quad (2.3)$$

In the presence of dilute point-like nonmagnetic impurities randomly substituted for the host atoms, the electronic self-energy can be adequately expressed in the SCTMA as

$$\Sigma^{R(A)} = n_i T^{R(A)} = \eta \gamma_0 \mp i \gamma_0, \quad (2.4)$$

where  $n_i$  is the impurity concentration,  $\gamma_0$  is the elastic scattering rate,  $\eta$  is a dimensional parameter, and  $\eta \gamma_0$  represents the shift of the chemical potential that can be absorbed in  $\mu$ . A use of Dyson's equation immediately yields the impurity-averaged one-particle Green's functions as

$$G_{\mathbf{k}}^{R(A)}(\epsilon) = \frac{1}{\epsilon - \epsilon_{\mathbf{k}} \pm i\gamma}, \quad (2.5)$$

where  $\gamma = \gamma_0 + \gamma_i$ , with  $\gamma_i$  the additional inelastic scattering rate induced by, e.g., the electron-electron and/or electron-phonon interactions. The intent of the introduction of  $\gamma_i$  is to investigate the dephasing effect on the QI process at finite temperatures that are assumed to be low enough so that  $\gamma_i(T) \ll \gamma_0$ . The DOS at the Fermi level of this disordered crystal can be readily calculated via

$$\rho_0 = -\frac{1}{\pi} \text{Im} \sum_{\mathbf{k}} G_{\mathbf{k}}^R = \frac{1}{\pi} \sum_{\mathbf{k}} \frac{\gamma}{\epsilon_{\mathbf{k}}^2 + \gamma^2}, \quad (2.6)$$

with  $G_{\mathbf{k}}^{R(A)} = G_{\mathbf{k}}^{R(A)}(0)$ . We consider, throughout this paper, the nearly half-filled band and the case of weak disorder, meaning that  $|\mu| \ll \gamma \ll t$ . Then, substituting Eq. (A1) in Appendix A into Eq. (2.6), we obtain

$$\rho_0 = \frac{1}{2\pi^2 a^2 t} \ln \frac{16t}{\gamma}. \quad (2.7)$$

The elastic scattering rate  $\gamma_0$  and the parameter  $\eta$  can be evaluated consistently by means of the  $T$ -matrix equation

$$T^{R(A)} = V + V \sum_{\mathbf{k}} G_{\mathbf{k}}^{R(A)} T^{R(A)}, \quad (2.8)$$

with  $V$  the strength of the impurity potential. A combination of Eq. (2.4) with Eq. (2.8) leads to

$$\gamma_0 = \frac{n_i}{\pi \rho_0 (1 + \eta^2)} \quad (2.9)$$

and

$$\eta = \frac{1}{\pi \rho_0 V}. \quad (2.10)$$

The Born limit corresponds to  $\eta^2 \gg 1$ , leading to

$$\gamma_0 = \pi n_i \rho_0 V^2; \quad (2.11)$$

and the unitary limit corresponds to  $\eta^2 \ll 1$ , yielding

$$\gamma_0 = \frac{n_i}{\pi\rho_0}. \quad (2.12)$$

Equations (2.11) and (2.12) have been pointed out in Ref. [17]. The Born and unitary approximations turns out to be two natural limits of the SCTMA, as the latter includes the effect of multiple-scattering process from the same impurity. As shown by Eq. (2.7),  $\rho_0$  has a large value in the weak-disorder case ( $\gamma$  is very small). From Eq. (2.10), it follows that the unitary condition can be satisfied for a finite impurity potential  $V$  and a low impurity concentration  $n_i$ .

### III. The diffusive modes

Since the QI effect in a disordered electron system results from the cooperon and diffuson, we first derive their expressions for a nearly half-filled band on the basis of the SCTMA described above. It is useful to note that the present system has two different kinds of symmetries. The first is the time-reversal symmetry described by

$$\epsilon_{-\mathbf{k}} = \epsilon_{\mathbf{k}}, \quad \mathbf{v}_{-\mathbf{k}} = -\mathbf{v}_{\mathbf{k}}, \quad (3.1)$$

and

$$G_{-\mathbf{k}}^{R(A)}(\epsilon) = G_{\mathbf{k}}^{R(A)}(\epsilon), \quad (3.2)$$

due to which the cooperon and diffuson have the same expressions. The other symmetry is given by

$$\epsilon_{\mathbf{Q}+\mathbf{k}} = -\epsilon_{\mathbf{k}} - 2\mu, \quad \mathbf{v}_{\mathbf{Q}+\mathbf{k}} = -\mathbf{v}_{\mathbf{k}}, \quad (3.3)$$

and

$$G_{\mathbf{Q}+\mathbf{k}}^{R(A)}(\epsilon) = -G_{\mathbf{k}}^{A(R)}(-\epsilon - 2\mu), \quad (3.4)$$

where  $\mathbf{Q}$  is one of four nesting vectors  $(\pm\pi/a, \pm\pi/a)$ . This symmetry is termed as particle-hole symmetry for the case of nested Fermi surface, and gives rise to the existence of the  $\pi$ -mode cooperon and diffuson in the unitary limit [18]. The ladder diagrams for the  $\pi$ -mode cooperon are depicted in Fig. 1. Unlike in the Born approximation used in the usual weak-localization theory, the dashed lines in Fig. 1 denote the  $T$ -matrix elements instead of the impurity potential  $V$ .

The equation for 0-mode cooperon in the retarded-advanced (RA) channel is given by

$$C(\mathbf{q}; \epsilon, \epsilon')^{RA} = W^{RA} + W^{RA} H(\mathbf{q}; \epsilon, \epsilon')^{RA} C(\mathbf{q}; \epsilon, \epsilon')^{RA}, \quad (3.5)$$

where

$$W^{RA} = n_i T^R T^A = \frac{\gamma_0}{\pi\rho_0} \quad (3.6)$$

and

$$H(\mathbf{q}; \epsilon, \epsilon')^{RA} = \sum_{\mathbf{k}} G_{\mathbf{q}-\mathbf{k}}^R(\epsilon) G_{\mathbf{k}}^A(\epsilon'). \quad (3.7)$$

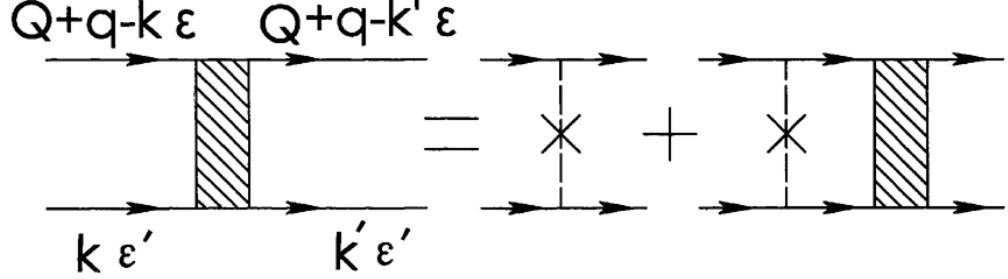


Figure 1: Ladder diagrams for the  $\pi$ -mode cooperon (shaded blocks). The dashed lines with crosses represent the  $T$ -matrix elements of the impurity scattering.  $\mathbf{Q}$  denotes the nesting vector. The  $\pi$ -mode diffuson corresponds to the similar diagrams with the arrows of the upper particle lines inverted. The diagrams for the 0-mode cooperon and diffuson can be obtained by setting  $\mathbf{Q} = 0$  in those of the diffusive  $\pi$ -modes.

Expanding the right side of Eq. (3.7) in powers of small  $\mathbf{q}$ ,  $\epsilon$ , and  $\epsilon'$ , we obtain

$$H(\mathbf{q}; \epsilon, \epsilon')^{RA} = \sum_{\mathbf{k}} \left[ G_{\mathbf{k}}^R G_{\mathbf{k}}^A + \frac{1}{2} \mathbf{q} \mathbf{q} : (\nabla \nabla G_{\mathbf{k}}^R) G_{\mathbf{k}}^A - \epsilon (G_{\mathbf{k}}^R)^2 G_{\mathbf{k}}^A - \epsilon' G_{\mathbf{k}}^R (G_{\mathbf{k}}^A)^2 \right]. \quad (3.8)$$

By carrying out a partial integral for the second term in the right side of Eq. (3.8), and using Eqs. (A2) and (A3), one gets

$$\begin{aligned} H(\mathbf{q}; \epsilon, \epsilon')^{RA} &= \sum_{\mathbf{k}} \left[ G_{\mathbf{k}}^R G_{\mathbf{k}}^A - \frac{1}{4} q^2 v_{\mathbf{k}}^2 (G_{\mathbf{k}}^R G_{\mathbf{k}}^A)^2 - \epsilon (G_{\mathbf{k}}^R)^2 G_{\mathbf{k}}^A - \epsilon' G_{\mathbf{k}}^R (G_{\mathbf{k}}^A)^2 \right] \\ &= \frac{\pi \rho_0}{2\gamma^2} \left[ 2\gamma - Dq^2 + i(\epsilon - \epsilon') \right], \end{aligned} \quad (3.9)$$

where the diffusion coefficient  $D$  is given by

$$D = \frac{2t}{\pi^2 \gamma \rho_0}. \quad (3.10)$$

Substituting Eqs. (3.6) and (3.9) into Eq. (3.5), and noting that the cooperon and diffuson have the same expressions, we obtain

$$C(\mathbf{q}; \epsilon, \epsilon')^{RA} = D(\mathbf{q}; \epsilon, \epsilon')^{RA} = \frac{2\gamma^2}{\pi \rho_0} \frac{1}{Dq^2 - i(\epsilon - \epsilon') + 2\gamma_i}. \quad (3.11)$$

The 0-mode cooperon and diffuson in the RR channel are non-singular, given by

$$C(\mathbf{q}; \epsilon, \epsilon')^{RR} = D(\mathbf{q}; \epsilon, \epsilon')^{RR} = -\frac{\gamma_0}{\pi \rho_0}. \quad (3.12)$$

Equations (3.11) and (3.12) are suitable for all values of  $\eta^2$  for the present crystal, while they were previously obtained in the Born limit for the free-electron model [5, 12, 13].

As in the disordered  $d$ -wave superconductors [14, 15], the  $\pi$ -mode cooperon and diffuson exist only in the unitary limit ( $\eta^2 \ll 1$ ). The equation for the  $\pi$ -mode cooperon in the RR channel is given by

$$C_\pi(\mathbf{q}; \epsilon, \epsilon')^{RR} = W^{RR} + W^{RR} H_\pi(\mathbf{q}; \epsilon, \epsilon')^{RR} C_\pi(\mathbf{q}; \epsilon, \epsilon')^{RR}, \quad (3.13)$$

where

$$W^{RR} = n_i (T^R)^2 = -\frac{\gamma_0}{\pi \rho_0}, \quad (\text{for } \eta^2 \ll 1) \quad (3.14)$$

and

$$H_\pi(\mathbf{q}; \epsilon, \epsilon')^{RR} = \sum_{\mathbf{k}} G_{\mathbf{Q}+\mathbf{q}-\mathbf{k}}^R(\epsilon) G_{\mathbf{k}}^R(\epsilon'). \quad (3.15)$$

Substituting Eq. (3.4) into Eq. (3.15), one can easily show that

$$H_\pi(\mathbf{q}; \epsilon, \epsilon')^{RR} = -\frac{\pi \rho_0}{2\gamma^2} \left[ 2\gamma - Dq^2 + i(\epsilon + \epsilon' + 2\mu) \right]. \quad (3.16)$$

A substitution of Eqs. (3.14) and (3.16) into Eq. (3.13) immediately yields

$$C_\pi(\mathbf{q}; \epsilon, \epsilon')^{RR(AA)} = D_\pi(\mathbf{q}; \epsilon, \epsilon')^{RR(AA)} = -\frac{2\gamma^2}{\pi \rho_0} \frac{1}{Dq^2 \mp i(\epsilon + \epsilon' + 2\mu) + 2\gamma_i}. \quad (3.17)$$

Similarly, we have

$$C_\pi(\mathbf{q}; \epsilon, \epsilon')^{RA} = D_\pi(\mathbf{q}; \epsilon, \epsilon')^{RA} = \frac{\gamma_0}{\pi \rho_0}. \quad (3.18)$$

We stress that the existence of the diffusive  $\pi$ -modes in RR or AA channel stems from the unitary condition and the particle-hole symmetry, similarly with the situation of disordered  $d$ -wave superconductors [14, 15].

It is instructive to compare the diffusive poles of the 0-mode and  $\pi$ -mode cooperons (diffusons) for zero temperature ( $\gamma_i = 0$ ). The 0-mode cooperon in RA channel is gapless, having a diffusive pole at a small energy difference and a small total momentum. The  $\pi$ -mode cooperon in RR or AA channel is gapped by the deviation from the perfectly-nested Fermi surface, with the diffusive pole at a total energy of  $-2\mu$  and a total momentum of the nesting vector. As will be shown below, the above difference between the 0-mode and  $\pi$ -mode cooperons (diffusons) makes the QI effect in the unitary limit significantly different from that of the Born limit.

## IV. QI correction to the conductivity

The above expressions for the cooperon and diffuson can be used to calculate the QI correction to the conductivity. The mean free path of the electrons is calculated as

$$l = \sqrt{\frac{D}{2\gamma}} = \frac{\sqrt{2}ta}{\gamma \ln(16t/\gamma)}, \quad (4.1)$$

which is much longer than the lattice constant  $a$  for the considered situation ( $\gamma \ll t$ ). Therefore, the usual quasiclassical approach can be used to calculate the conductivity. The electrical

current density is expressed by  $\mathbf{j} = -e \sum_{\mathbf{k}\sigma} \mathbf{v}_{\mathbf{k}} c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\sigma}$ , with  $c_{\mathbf{k}\sigma}^\dagger$  and  $c_{\mathbf{k}\sigma}$  standing for the creation and annihilation operators of electrons, respectively. According to the Kubo formula, the dc conductivity related with the impurity scattering is given by [19]

$$\sigma = \sigma^{RA} + \sigma^{RR}, \quad (4.2)$$

with

$$\begin{aligned} \sigma^{RA(RR)} &= \pm \lim_{\omega \rightarrow 0} \frac{1}{\omega} \int_{-\infty}^{+\infty} \frac{d\epsilon}{2\pi} [f(\epsilon - \omega) - f(\epsilon)] \text{Re}\Pi(\epsilon, \epsilon - \omega)^{RA(RR)} \\ &= \pm \int_{-\infty}^{+\infty} \frac{d\epsilon}{2\pi} \frac{\text{Re}\Pi(\epsilon, \epsilon)^{RA(RR)}}{4T \cosh^2(\epsilon/2T)}, \end{aligned} \quad (4.3)$$

where  $f(\epsilon)$  denotes the Fermi distribution function, and  $\Pi(\epsilon, \epsilon')^{RA}$  and  $\Pi(\epsilon, \epsilon')^{RR}$  represent the electrical current-current correlation functions in RA and RR channels, respectively. In this paper, only the case of

$$\gamma_i \ll \gamma \text{ and } T \ll \gamma, \quad (4.4)$$

is considered.

The Drud conductivity corresponds to the contribution of the “bare bubble” diagram, which is given by (for  $T = 0$ )

$$\sigma_0 = \frac{1}{2\pi} \text{Re}\Pi(0, 0)_0^{RA} = \frac{e^2}{2\pi} \sum_{\mathbf{k}} v_{\mathbf{k}}^2 G_{\mathbf{k}}^R G_{\mathbf{k}}^A. \quad (4.5)$$

Using Eq. (A3), we obtain

$$\sigma_0 = 2e^2 \rho_0 D, \quad (4.6)$$

meaning that the Einstein relation is also suitable for the present nearly half-filled band.

## A. Correlation functions responsible for the QI effect

The correlation functions responsible for the QI effect can be calculated by the polarization diagrams with cooperon and diffuson. As in the usual weak-localization theory, the 0-mode diffuson has no contribution to the QI effect. The contribution of 0-mode cooperon to the conductivity is represented by Fig. 2(c) with  $\mathbf{Q} = 0$  (the maximally-crossed diagrams), and the corresponding correlation function can be evaluated as

$$\begin{aligned} \Pi(\epsilon, \epsilon')_{\text{coop}}^{RA} &= e^2 \sum_{\mathbf{k}\mathbf{q}} \mathbf{v}_{\mathbf{k}} \cdot \mathbf{v}_{-\mathbf{k}} G_{\mathbf{k}}^R G_{-\mathbf{k}}^R G_{\mathbf{k}}^A G_{-\mathbf{k}}^A C(\mathbf{q}; \epsilon, \epsilon')^{RA} \\ &= -e^2 \sum_{\mathbf{k}\mathbf{q}} v_{\mathbf{k}}^2 (G_{\mathbf{k}}^R G_{\mathbf{k}}^A)^2 C(\mathbf{q}; \epsilon, \epsilon')^{RA}. \end{aligned} \quad (4.7)$$

Substituting Eq. (3.11) into Eq. (4.7), and using Eq. (A3), we get

$$\Pi(\epsilon, \epsilon')_{\text{coop}}^{RA} = - \sum_{\mathbf{q}} \frac{4e^2 D}{Dq^2 - i(\epsilon - \epsilon') + 2\gamma_i}, \quad (4.8)$$



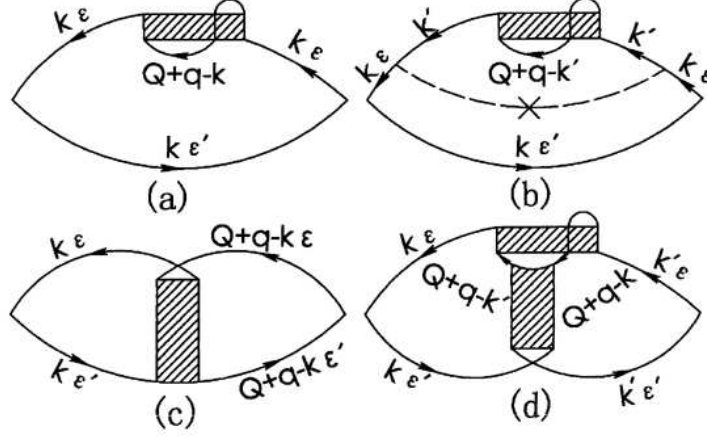


Figure 2: Leading conductivity diagrams with  $\pi$ -mode cooperon (shaded blocks).

which is valid for all values of  $\eta^2$ .

Besides the 0-mode cooperon, the  $\pi$ -mode cooperon and diffuson also have contributions to the conductivity in the unitary limit, and the leading polarization diagrams are shown in Figs. 2 and 3, respectively [18]. These diagrams can be generated using the conserving approximation, as in the theory of disordered interacting electron systems [5, 20, 21]. In Appendix B, we show that the RR-correlation functions from the diagrams in Figs. 2 and 3 contribute vanishing corrections to the dc conductivity, similarly with the case of disordered interacting electrons [22]. This feature is a manifestation of the conserving law for the number of particles. Thus, only the RA-correlation functions are calculated in this section.

The non-trivial contributions of the  $\pi$ -mode cooperon to the RA-correlation function result from Figs. 2(a) and 2(b). They are expressed, respectively, as

$$\begin{aligned}\Pi(\epsilon, \epsilon')_{2a}^{RA} &= e^2 \sum_{\mathbf{k} \mathbf{q}} \mathbf{v}_{\mathbf{k}} \cdot \mathbf{v}_{\mathbf{k}} (G_{\mathbf{k}}^R)^2 G_{\mathbf{Q}-\mathbf{k}}^R G_{\mathbf{k}}^A C_{\pi}(\mathbf{q}; \epsilon, \epsilon)^{RR} \\ &= -e^2 \sum_{\mathbf{k} \mathbf{q}} v_{\mathbf{k}}^2 (G_{\mathbf{k}}^R G_{\mathbf{k}}^A)^2 C_{\pi}(\mathbf{q}; \epsilon, \epsilon)^{RR}\end{aligned}\quad (4.9)$$

and

$$\begin{aligned}\Pi(\epsilon, \epsilon')_{2b}^{RA} &= e^2 \sum_{\mathbf{k} \mathbf{k}' \mathbf{q}} \mathbf{v}_{\mathbf{k}} \cdot \mathbf{v}_{\mathbf{k}'} (G_{\mathbf{k}}^R G_{\mathbf{k}'}^R)^2 G_{\mathbf{Q}-\mathbf{k}'}^R G_{\mathbf{k}}^A W^{RR} C_{\pi}(\mathbf{q}; \epsilon, \epsilon)^{RR} \\ &= -e^2 \sum_{\mathbf{k} \mathbf{k}' \mathbf{q}} v_{\mathbf{k}}^2 (G_{\mathbf{k}}^R G_{\mathbf{k}'}^R)^2 G_{\mathbf{k}}^A G_{\mathbf{k}'}^A W^{RR} C_{\pi}(\mathbf{q}; \epsilon, \epsilon)^{RR},\end{aligned}\quad (4.10)$$

where the particle-hole symmetry shown by Eq. (3.4) has been used. Substituting Eqs. (3.14) and (3.17) into Eqs. (4.9) and (4.10), and using Eqs. (A2) and (A3), one can readily show that

$$\Pi(\epsilon, \epsilon')_{2a}^{RA} = 2\Pi(\epsilon, \epsilon')_{2b}^{RA} = \sum_{\mathbf{q}} \frac{4e^2 D}{Dq^2 - i2(\epsilon + \mu) + 2\gamma_i}. \quad (4.11)$$

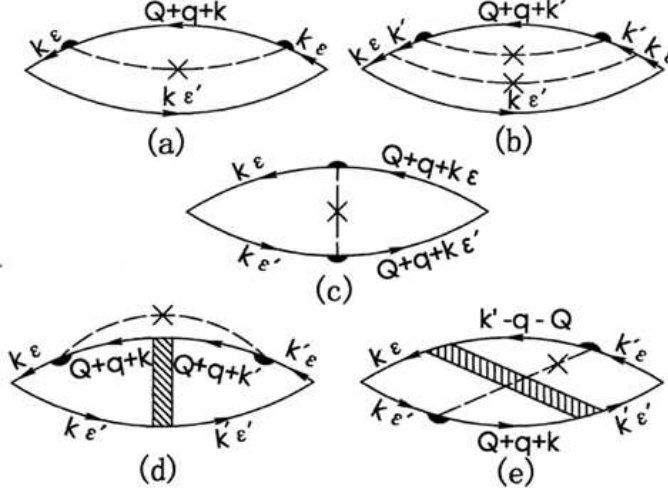


Figure 3: Leading conductivity diagrams with  $\pi$ -mode diffusion (shaded blocks). The dark parts denote the vertex corrections to the impurity scattering due to the  $\pi$ -mode diffuson.

The contributions of Figs. 3(a)–3(c) to the RA-correlation function can be expressed, respectively, as

$$\begin{aligned}\Pi(\epsilon, \epsilon')_{3a}^{RA} &= e^2 \sum_{\mathbf{k}\mathbf{q}} \mathbf{v}_{\mathbf{k}} \cdot \mathbf{v}_{\mathbf{k}} (G_{\mathbf{k}}^R)^2 G_{\mathbf{Q}+\mathbf{k}}^R G_{\mathbf{k}}^A W^{RR} \Lambda^R(\mathbf{q}, \epsilon)^2 \\ &= -e^2 \sum_{\mathbf{k}\mathbf{q}} v_{\mathbf{k}}^2 (G_{\mathbf{k}}^R G_{\mathbf{k}}^A)^2 W^{RR} \Lambda^R(\mathbf{q}, \epsilon)^2,\end{aligned}\quad (4.12)$$

$$\begin{aligned}\Pi(\epsilon, \epsilon')_{3b}^{RA} &= e^2 \sum_{\mathbf{k}\mathbf{k}'\mathbf{q}} \mathbf{v}_{\mathbf{k}} \cdot \mathbf{v}_{\mathbf{k}'} (G_{\mathbf{k}}^R G_{\mathbf{k}'}^R)^2 G_{\mathbf{Q}+\mathbf{k}'}^R G_{\mathbf{k}}^A (W^{RR})^2 \Lambda^R(\mathbf{q}, \epsilon)^2 \\ &= -e^2 \sum_{\mathbf{k}\mathbf{k}'\mathbf{q}} v_{\mathbf{k}}^2 (G_{\mathbf{k}}^R G_{\mathbf{k}'}^R)^2 G_{\mathbf{k}}^A G_{\mathbf{k}'}^A (W^{RR})^2 \Lambda^R(\mathbf{q}, \epsilon)^2,\end{aligned}\quad (4.13)$$

and

$$\begin{aligned}\Pi(\epsilon, \epsilon')_{3c}^{RA} &= e^2 \sum_{\mathbf{k}\mathbf{q}} \mathbf{v}_{\mathbf{k}} \cdot \mathbf{v}_{\mathbf{Q}+\mathbf{k}} G_{\mathbf{k}}^R G_{\mathbf{Q}+\mathbf{k}}^R G_{\mathbf{Q}+\mathbf{k}}^A G_{\mathbf{k}}^A W^{RA} \Lambda^R(\mathbf{q}, \epsilon) \Lambda^A(\mathbf{q}, \epsilon') \\ &= -e^2 \sum_{\mathbf{k}\mathbf{q}} v_{\mathbf{k}}^2 (G_{\mathbf{k}}^R G_{\mathbf{k}}^A)^2 W^{RA} \Lambda^R(\mathbf{q}, \epsilon) \Lambda^A(\mathbf{q}, \epsilon'),\end{aligned}\quad (4.14)$$

where  $\Lambda^{R(A)}(\mathbf{q}, \epsilon)$  represent the vertex functions of the impurity scattering due to the  $\pi$ -mode diffusion, as shown by Fig. 4. Obviously, the retarded vertex function is given by

$$\Lambda^R(\mathbf{q}, \epsilon) = 1 + D_{\pi}(\mathbf{q}; \epsilon, \epsilon)^{RR} H_{\pi}(\mathbf{q}; \epsilon, \epsilon)^{RR}. \quad (4.15)$$

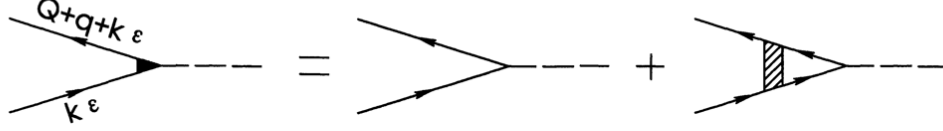


Figure 4: Diagrams for the vertex correction to the impurity scattering. The shaded block stands for the  $\pi$ -mode diffusion.

A substitution of Eqs. (3.16) and (3.17) into Eq. (4.15) immediately yields

$$\Lambda^{R(A)}(\mathbf{q}, \epsilon) = \frac{2\gamma}{Dq^2 \mp i2(\epsilon + \mu) + 2\gamma_i}, \quad (4.16)$$

for  $Dq^2 \ll \gamma$  and  $|\epsilon| \ll \gamma$ . Substituting Eqs. (3.6), (3.14), and (4.16) into Eqs. (4.12)–(4.14), and using Eqs. (A2) and (A3), we obtain

$$\Pi(\epsilon, \epsilon')_{3a}^{RA} = 2\Pi(\epsilon, \epsilon')_{3b}^{RA} = \sum_{\mathbf{q}} \frac{8e^2 D\gamma}{[Dq^2 - i2(\epsilon + \mu) + 2\gamma_i]^2} \quad (4.17)$$

and

$$\Pi(\epsilon, \epsilon')_{3c}^{RA} = - \sum_{\mathbf{q}} \frac{8e^2 D\gamma}{[Dq^2 - i2(\epsilon + \mu) + 2\gamma_i][Dq^2 + i2(\epsilon' + \mu) + 2\gamma_i]}. \quad (4.18)$$

Similarly, the RA-correlation functions corresponding to Figs. 3(d) and 3(e) are expressed by

$$\Pi(\epsilon, \epsilon')_{3d}^{RA} = e^2 \sum_{\mathbf{k}\mathbf{k}'\mathbf{q}} \mathbf{v}_{\mathbf{k}} \cdot \mathbf{v}_{\mathbf{k}'} G_{\mathbf{k}}^R G_{\mathbf{k}'}^R G_{\mathbf{Q}+\mathbf{q}+\mathbf{k}}^R G_{\mathbf{Q}+\mathbf{q}+\mathbf{k}'}^R G_{\mathbf{k}}^A G_{\mathbf{k}'}^A W^{RR} \Lambda^R(\mathbf{q}, \epsilon)^2 D_{\pi}(\mathbf{q}; \epsilon, \epsilon')^{RA} \quad (4.19)$$

and

$$\Pi(\epsilon, \epsilon')_{3e}^{RA} = e^2 \sum_{\mathbf{k}\mathbf{k}'\mathbf{q}} \mathbf{v}_{\mathbf{k}} \cdot \mathbf{v}_{\mathbf{k}'} G_{\mathbf{k}}^R G_{\mathbf{k}'}^R G_{-\mathbf{Q}-\mathbf{q}+\mathbf{k}'}^R G_{\mathbf{Q}+\mathbf{q}+\mathbf{k}}^A G_{\mathbf{k}}^A G_{\mathbf{k}'}^A W^{RA} \Lambda^R(\mathbf{q}, \epsilon) \Lambda^A(\mathbf{q}, \epsilon') D_{\pi}(\mathbf{q}; \epsilon, \epsilon')^{RA}. \quad (4.20)$$

For small  $\mathbf{q}$  ( $Dq^2 \ll \gamma$ ), Eqs. (4.19) and (4.20) can be changed as

$$\Pi(\epsilon, \epsilon')_{3d}^{RA} = \frac{e^2}{4} \sum_{\mathbf{k}\mathbf{k}'\mathbf{q}} q^2 v_{\mathbf{k}}^2 v_{\mathbf{k}'}^2 G_{\mathbf{k}}^R G_{\mathbf{k}'}^R (G_{\mathbf{k}}^A G_{\mathbf{k}'}^A)^3 W^{RR} \Lambda^R(\mathbf{q}, \epsilon)^2 D_{\pi}(\mathbf{q}; \epsilon, \epsilon')^{RA} \quad (4.21)$$

and

$$\Pi(\epsilon, \epsilon')_{3e}^{RA} = -\frac{e^2}{4} \sum_{\mathbf{k}\mathbf{k}'\mathbf{q}} q^2 v_{\mathbf{k}}^2 v_{\mathbf{k}'}^2 G_{\mathbf{k}}^A G_{\mathbf{k}'}^R (G_{\mathbf{k}}^R G_{\mathbf{k}'}^A)^3 W^{RA} \Lambda^R(\mathbf{q}, \epsilon) \Lambda^A(\mathbf{q}, \epsilon') D_{\pi}(\mathbf{q}; \epsilon, \epsilon')^{RA}. \quad (4.22)$$

Substituting Eqs. (3.6), (3.14), (3.18), and (4.16) into Eqs. (4.21) and (4.22), and using Eqs. (A2) and (A3), we obtain

$$\Pi(\epsilon, \epsilon')_{3d}^{RA} = - \sum_{\mathbf{q}} \frac{e^2 D^2 q^2}{[Dq^2 - i2(\epsilon + \mu) + 2\gamma_i]^2}, \quad (4.23)$$

and

$$\Pi(\epsilon, \epsilon')_{3e}^{RA} = - \sum_{\mathbf{q}} \frac{e^2 D^2 q^2}{[Dq^2 - i2(\epsilon + \mu) + 2\gamma_i][Dq^2 + i2(\epsilon' + \mu) + 2\gamma_i]}. \quad (4.24)$$

The expressions for the correlation function, Eqs. (4.8), (4.11), (4.17), (4.18), (4.23), and (4.24), are the main results we obtain in this subsection, and will be used to calculate the dependence of the conductivity on the temperature or sample size. It is worthy to point out that Eqs. (4.17) and (4.18) are more divergent than the others. The contributions of Figs. 3(a)–3(c) to the conductivity have been presented in Ref. [18] for the case of  $\mu = T = 0$ . In the following calculations of the conductivity, we will neglect the terms of the order  $e^2/\pi^2$ , which is much smaller than the Drud one, i.e.,  $e^2/\pi^2\sigma_0 = \gamma/4t \ll 1$ .

## B. Temperature dependence of the conductivity

In non-unitary cases, only the 0-mode cooperon contributes to the QI effect. Substituting Eq. (4.8) into Eq. (4.3), and noting that the upper cutoff of  $q$  is set to be  $1/l$ , we can readily obtain

$$\delta\sigma(T) = \sigma(T)_{\text{coop}}^{RA} = -\frac{e^2}{2\pi^2} \ln \frac{\gamma}{\gamma_i}, \quad (4.25)$$

where we have used

$$\int_0^{+\infty} \frac{dx}{\cosh^2 x} = 1.$$

Although Eq. (4.25) was obtained within the Born approximation in the usual weak-localization theory, we show that it is valid for all values of  $\eta^2$  in the present system. The logarithmic weak-localization effect results from the singular backscattering due to the time-reversal symmetry  $v_{-\mathbf{k}} = -v_{\mathbf{k}}$ .

In the unitary limit, the  $\pi$ -mode cooperon and diffuson also contribute to the conductivity. Substituting Eq. (4.11) into Eq. (4.3), and completing the integral over  $\mathbf{q}$ , we get

$$\sigma(T)_{\pi\text{-coop}}^{RA} = 2\sigma(T)_{2a}^{RA} + 2\sigma(T)_{2b}^{RA} = \frac{3e^2}{8\pi^2} \int_{-\infty}^{+\infty} \frac{dx}{\cosh^2 x} \ln \frac{\gamma^2}{(2Tx + \mu)^2 + \gamma_i^2}, \quad (4.26)$$

yielding

$$\sigma(T)_{\pi\text{-coop}}^{RA} = \frac{3e^2}{2\pi^2} \times \begin{cases} \ln(\gamma/\gamma_i), & \text{if } \gamma_i \gg T, |\mu|, \\ \ln(\gamma/T), & \text{if } T \gg \gamma_i, |\mu|, \\ \ln(\gamma/|\mu|), & \text{if } |\mu| \gg T, \gamma_i. \end{cases} \quad (4.27)$$

Equation (4.27) indicates that the  $\pi$ -mode cooperon contributes a logarithmic anti-localization correction to the conductivity. This weak anti-localization correction occurs as the result of the singular forward-scattering processes shown by Figs. 2(a) and 2(b).

Similarly, by means of Eqs. (4.17), (4.18), (4.23), and (4.24), we can show that

$$\begin{aligned} \sigma(T)_{3a}^{RA} &= 2\sigma(T)_{3b}^{RA} = \frac{e^2}{4\pi^2} \int_{-\infty}^{+\infty} \frac{dx}{\cosh^2 x} \frac{\gamma\gamma_i}{(2Tx + \mu)^2 + \gamma_i^2} \\ &= \frac{e^2}{2\pi^2} \times \begin{cases} \gamma/\gamma_i, & \text{if } \gamma_i \gg T, |\mu|, \\ \pi\gamma/4T, & \text{if } T \gg \gamma_i, |\mu|, \\ \gamma\gamma_i/\mu^2, & \text{if } |\mu| \gg T, \gamma_i, \end{cases} \end{aligned} \quad (4.28)$$

$$\begin{aligned}
\sigma(T)_{3c}^{RA} &= -\frac{e^2}{4\pi^2} \int_{-\infty}^{+\infty} \frac{dx}{\cosh^2 x} \frac{\gamma}{2Tx + \mu} \arctan \frac{2Tx + \mu}{\gamma_i} \\
&= -\frac{e^2}{2\pi^2} \times \begin{cases} \gamma/\gamma_i, & \text{if } \gamma_i \gg T, |\mu|, \\ \pi\gamma/4T, & \text{if } T \gg \gamma_i, |\mu|, \\ \pi\gamma/2|\mu|, & \text{if } |\mu| \gg T, \gamma_i, \end{cases}
\end{aligned} \tag{4.29}$$

and

$$\begin{aligned}
\sigma(T)_{3d}^{RA} &= \sigma(T)_{3e}^{RA} = -\frac{e^2}{32\pi^2} \int_{-\infty}^{+\infty} \frac{dx}{\cosh^2 x} \ln \frac{\gamma^2}{(2Tx + \mu)^2 + \gamma_i^2} \\
&= -\frac{e^2}{8\pi^2} \times \begin{cases} \ln(\gamma/\gamma_i), & \text{if } \gamma_i \gg T, |\mu|, \\ \ln(\gamma/T), & \text{if } T \gg \gamma_i, |\mu|, \\ \ln(\gamma/|\mu|), & \text{if } |\mu| \gg T, \gamma_i, \end{cases}
\end{aligned} \tag{4.30}$$

where we have used

$$\lim_{x_0 \rightarrow 0^+} \int_{-\infty}^{+\infty} \frac{dx}{\cosh^2 x} \frac{x_0}{x^2 + x_0^2} = \pi,$$

and

$$\lim_{x_0 \rightarrow 0^+} \int_{-\infty}^{+\infty} \frac{dx}{\cosh^2 x} \frac{1}{x} \arctan \frac{x}{x_0} = \pi.$$

Therefore, the contribution from the  $\pi$ -mode diffuson is given by

$$\begin{aligned}
\sigma(T)_{\pi\text{-diff}}^{RA} &= 2\sigma(T)_{3a}^{RA} + 2\sigma(T)_{3b}^{RA} + \sigma(T)_{3c}^{RA} + 2\sigma(T)_{3d}^{RA} + 2\sigma(T)_{3e}^{RA} \\
&= \frac{e^2}{2\pi^2} \times \begin{cases} 2\gamma/\gamma_i - \ln(\gamma/\gamma_i), & \text{if } \gamma_i \gg T, |\mu|, \\ \pi\gamma/2T - \ln(\gamma/T), & \text{if } T \gg \gamma_i, |\mu|, \\ -\pi\gamma/2|\mu| - \ln(\gamma/|\mu|), & \text{if } |\mu| \gg T, \gamma_i. \end{cases}
\end{aligned} \tag{4.31}$$

The dominant power-law terms in Eq. (4.31) come from the more divergent contributions of Figs. 3(a)–3(c). While Figs. 3(a) and 3(b) correspond to singular forward scatterings, Fig. 3(c) describes a singular backscattering process due to the particle-hole symmetry  $v_{\mathbf{Q}+\mathbf{k}} = -v_{\mathbf{k}}$ .

The total QI correction to the dc conductivity in the unitary limit is given by

$$\delta\sigma_U(T) = \sigma(T)_{\text{coop}}^{RA} + \sigma(T)_{\pi\text{-coop}}^{RA} + \sigma(T)_{\pi\text{-diff}}^{RA},$$

having the following temperature behavior:

$$\delta\sigma_U(T) \sim \frac{e^2}{2\pi^2} \times \begin{cases} 2\gamma/\gamma_i, & \text{if } \gamma_i \gg T, |\mu|, \\ \pi\gamma/2T, & \text{if } T \gg \gamma_i, |\mu|, \\ -\ln(\gamma/\gamma_i), & \text{if } |\mu| \gg T, \gamma_i. \end{cases} \tag{4.32}$$

At low temperatures, it is reasonable to assume that  $\gamma_i = \gamma(T/T_0)^p$  with  $p > 1$  for the electron-electron or electron-phonon interactions [5, 12]. Then we get

$$\delta\sigma_U(T) \sim \frac{e^2}{2\pi^2} \times \begin{cases} 2(T_0/T)^p, & \text{if } \gamma_i \gg T, |\mu|, \\ \pi\gamma/2T, & \text{if } T \gg \gamma_i, |\mu|, \\ -p\ln(T_0/T), & \text{if } |\mu| \gg T, \gamma_i. \end{cases} \tag{4.33}$$

Equation (4.33) indicates that the QI correction to the dc conductivity in the unitary limit is a non-monotonic function of the temperature. The temperature dependence of the conductivity changes from power-law anti-localization behaviors to a logarithmic localization one with decreasing the temperature, where the crossover occurs at  $T \sim |\mu|$ .

The temperature behavior of the conductivity can be understood by analyzing the dephasing effects on the QI processes. While the QI process related with the 0-mode cooperon is dephased by the inelastic scattering, both the inelastic scattering and the thermal fluctuation are the dephasing factors of the QI effects resulting from the diffusive  $\pi$ -modes. This is because the diffusive 0-modes and  $\pi$ -modes have different diffusive poles. In the region of  $\min\{\gamma_i, T\} \gg |\mu|$ , the QI effect results dominantly from the contributions of diagrams 3(a)–3(c) that contain the  $\pi$ -mode diffuson. In the case of  $\gamma_i \gg T, |\mu|$ , the inelastic scattering is the main dephasing mechanism. For the situation of  $T \gg \gamma_i, |\mu|$ , however, the thermal fluctuation becomes the dominant dephasing factor. If the deviation from the nesting Fermi surface is large enough so that  $|\mu| \gg \gamma_i, T$ , the contributions of the diffusive  $\pi$ -modes are suppressed, and the QI effect stems only from the 0-mode cooperon that is dephased by the inelastic scattering.

### C. Sample-size dependence of the conductivity

At zero temperature ( $T = \gamma_i = 0$ ), the conductivity formula, Eq.(4.3), reduces to be

$$\sigma(L)^{RA(RR)} = \pm \frac{1}{2\pi} \text{Re}\Pi(0, 0)^{RA(RR)}. \quad (4.34)$$

For a finite system, the upper and lower cutoffs of  $q$  in the correlation functions are set to be  $1/l$  and  $1/L$ , respectively.

In non-unitary cases, the conductivity is easily shown to be subject to a logarithmic weak-localization correction as

$$\delta\sigma(L) = \sigma(L)_{\text{coop}}^{RA} = -\frac{e^2}{2\pi^2} \ln \frac{\gamma}{\gamma_L}, \quad (4.35)$$

which is in agreement with Eq. (1.1).

In the unitary limit, the  $\pi$ -mode cooperon is shown to yield a logarithmic anti-localization correction to the conductivity as

$$\begin{aligned} \sigma(L)_{\pi\text{-coop}}^{RA} &= 2\sigma(L)_{2a}^{RA} + 2\sigma(L)_{2b}^{RA} = \frac{3e^2}{4\pi^2} \ln \frac{\gamma^2}{\gamma_L^2 + \mu^2} \\ &= \frac{3e^2}{2\pi^2} \times \begin{cases} \ln(\gamma/\gamma_L), & \text{if } \gamma_L \gg |\mu|, \\ \ln(\gamma/|\mu|), & \text{if } |\mu| \gg \gamma_L. \end{cases} \end{aligned} \quad (4.36)$$

Similarly, substituting Eqs. (4.17), (4.18), (4.23), and (4.24) into Eq. (4.34), we can readily show that

$$\sigma(L)_{3a}^{RA} = 2\sigma(L)_{3b}^{RA} = \frac{e^2}{2\pi^2} \frac{\gamma\gamma_L}{\gamma_L^2 + \mu^2}, \quad (4.37)$$

$$\sigma(L)_{3c}^{RA} = -\frac{e^2}{2\pi^2} \frac{\gamma}{\mu} \arctan \frac{\mu}{\gamma_L}, \quad (4.38)$$

and

$$\sigma(L)_{3d}^{RA} = \sigma(L)_{3e}^{RA} = -\frac{e^2}{16\pi^2} \ln \frac{\gamma^2}{\gamma_L^2 + \mu^2}. \quad (4.39)$$

Thus, we get

$$\begin{aligned} \sigma(L)_{\pi\text{-diff}}^{RA} &= 2\sigma(L)_{3a}^{RA} + 2\sigma(L)_{3b}^{RA} + \sigma(L)_{3c}^{RA} + 2\sigma(L)_{3d}^{RA} + 2\sigma(L)_{3e}^{RA} \\ &= \frac{e^2}{2\pi^2} \times \begin{cases} 2\gamma/\gamma_L - \ln(\gamma/\gamma_L), & \text{if } \gamma_L \gg |\mu|, \\ -\pi\gamma/2|\mu| - \ln(\gamma/|\mu|), & \text{if } |\mu| \gg \gamma_L. \end{cases} \end{aligned} \quad (4.40)$$

The dominant power-law terms in above equation are also from the contributions of Figs. 3(a)–3(c).

As a result, the total QI correction to the conductivity in the unitary limit is given by

$$\begin{aligned} \delta\sigma_U(L) &= \sigma(L)_{\text{coop}}^{RA} + \sigma(L)_{\pi\text{-coop}}^{RA} + \sigma(L)_{\pi\text{-diff}}^{RA} \\ &\sim \frac{e^2}{2\pi^2} \times \begin{cases} 2\gamma/\gamma_L, & \text{if } \gamma_L \gg |\mu|, \\ -\ln(\gamma/\gamma_L), & \text{if } |\mu| \gg \gamma_L. \end{cases} \end{aligned} \quad (4.41)$$

It is instructive to define a characteristic length as

$$\xi = \sqrt{\frac{D}{2|\mu|}} = l\sqrt{\frac{\gamma}{|\mu|}} \gg l. \quad (4.42)$$

Then Eq. (4.41) can be expressed as

$$\delta\sigma_U(L) \sim \frac{e^2}{\pi^2} \times \begin{cases} L^2/l^2, & \text{if } L \ll \xi, \\ -\ln(L/l), & \text{if } L \gg \xi, \end{cases} \quad (4.43)$$

which is also a non-monotonic function of the sample size. For a small sample ( $L \ll \xi$ ), the conductivity increases with  $L^2$  due to the contributions of diagrams 3(a)–3(c) containing the  $\pi$ -mode diffuson. In the case of  $L \gg \xi$ , the QI corrections from the diffusive  $\pi$ -modes are suppressed, and the usual weak-localization effect remains due to the contribution of 0-mode coooperon.

## V. Conclusions

Using the SCTMA, we have investigated theoretically the effect of multiple-scattering of impurities on the QI of electrons in 2D crystals with nearly half-filled bands. The particle-hole symmetry, together with the van Hove singularity at the band center, is shown to alter qualitatively the results predicted by the usual weak-localization theory.

For a definite impurity potential  $V$ , the large value of  $\rho_0$  ( the DOS at the Fermi level) can drive the system into the unitary limit with decreasing the impurity concentration  $n_i$ . While the usual 0-mode coooperon and diffuson exist in general situations (including the Born and unitary limits), the additional  $\pi$ -mode coooperon and diffuson appear only in the unitary limit due to the existence of the particle-hole symmetry, similarly with the situation of disordered  $d$ -wave superconductors [14, 15]. The diffusive 0-modes are gapless, and of diffusive poles at small energy difference. On the contrary, the diffusive  $\pi$ -modes are gapped by the deviation

from perfectly-nested Fermi surface measured by  $|\mu|$ , with the diffusive poles at the total energy of  $-2\mu$ .

In non-unitary cases, only the 0-mode cooperon contributes to the QI effect, yielding a weak-localization correction to the conductivity described by Eq. (1.1). Upon approaching the unitary limit, there exist some additional conductivity diagrams containing the  $\pi$ -mode cooperon or  $\pi$ -mode diffuson, giving rise to additional corrections to the conductivity. In the case of small  $|\mu|$ , the conductivity in the unitary limit is subject to power-law anti-localization corrections, which come predominantly from the contribution of the  $\pi$ -mode diffuson. However, this anti-localization effect is suppressed by large deviation from the nesting case, and the usual logarithmic weak-localization effect survives due to the 0-mode cooperon. As a result, the dc conductivity in the unitary limit becomes a non-monotonic function of the temperature or the sample size.

The calculations of the conductivity at finite temperatures reveal that, both the inelastic scattering and the thermal fluctuation are the dephasing factors in the QI processes relevant to the diffusive  $\pi$ -modes. In the region of  $\gamma_i \gg T, |\mu|$ , the inelastic scattering is the dominant dephasing mechanism. For the case of  $T \gg \gamma_i, |\mu|$ , the QI effect is mainly dephased by the thermal fluctuation. This unique feature is related with the diffusive poles of the  $\pi$ -mode cooperon and diffuson.

At zero temperature, there exists a characteristic length  $\xi$  defined by Eq. (4.42). While the dc conductivity is subject to a logarithmic weak-localization correction for large samples ( $L \gg \xi$ ), it increases as  $L^2$  in the region of  $L \ll \xi$ . This non-monotonic behavior of the conductivity might be instructive for the numerical study of the present system, as the latter is usually carried out for a finite sample. For a perfectly-nested Fermi surface ( $\xi \rightarrow \infty$ ), the anti-localization effect suggests the existence of extended states at the band center. We note that the non-localized states were also found in the center of the band of 2D Anderson model with purely off-diagonal disorder [6–10]. Numerical studies of these systems revealed that the localization length diverges at the band center [8, 9]. The present analytical work tries to put some physical insights into the understanding of the delocalization effect in the considered system.

Since the diffusive  $\pi$ -modes introduce some additional conductivity diagrams, they are expected to play non-trivial roles in other transport properties such as the magnetoresistance and the Hall effect in this system. How these new diagrams affect the quasiparticle localization in disordered  $d$ -wave superconductors is also an interesting and open problem.

## Acknowledgment

This work was supported by the National Natural Science Foundation of China under Grants No. 10274008 and No. 10174011, and the Jiangsu-Province Natural Science Foundation of China under Grant No. BK2002050. DYX would like to acknowledge the support of Grant No. G19980614 for State Key Programs for Basic Research of China.

## Appendix A: Some useful mathematical formulas

In this appendix, we will prove the following mathematical formulas

$$\sum_{\mathbf{k}} \frac{1}{\epsilon_{\mathbf{k}}^2 + \gamma^2} = \frac{1}{2\pi a^2 t \gamma} \ln \frac{16t}{\gamma}, \quad (\text{A1})$$



$$\sum_{\mathbf{k}} (G_{\mathbf{k}}^R)^m (G_{\mathbf{k}}^A)^n = 2\pi\rho_0 \frac{(m+n-2)!}{(m-1)!(n-1)!} \times \frac{i^{n-m}}{(2\gamma)^{m+n-1}}, \quad (\text{A2})$$

$$\sum_{\mathbf{k}} v_{\mathbf{k}}^2 (G_{\mathbf{k}}^R)^m (G_{\mathbf{k}}^A)^n = 4\pi\rho_0 D \frac{(m+n-2)!}{(m-1)!(n-1)!} \times \frac{i^{n-m}}{(2\gamma)^{m+n-2}}, \quad (\text{A3})$$

where  $m, n \geq 1$ . Although Eqs. (A2) and (A3) are exactly the same as those of the free-electron model, we shall show that they are also valid for the present nearly half-filled band.

The summation over  $\mathbf{k}$  in Eq. (A1) can be replaced by an integral, i.e.,

$$\sum_{\mathbf{k}} \frac{1}{\epsilon_{\mathbf{k}}^2 + \gamma^2} = \int_{-4t}^{4t} d\epsilon \frac{\rho_{\text{cl}}(\epsilon)}{\epsilon^2 + \gamma^2}. \quad (\text{A4})$$

In the case of  $|\mu| \ll \gamma \ll t$  under considered, the density of states  $\rho_{\text{cl}}(\epsilon)$  in Eq. (A4) can be replaced by the expression near the band center given by Eq. (2.3), yielding

$$\begin{aligned} \sum_{\mathbf{k}} \frac{1}{\epsilon_{\mathbf{k}}^2 + \gamma^2} &\approx \frac{1}{2\pi^2 a^2 t} \int_{-\infty}^{+\infty} \frac{d\epsilon}{\epsilon^2 + \gamma^2} \ln \frac{16t}{|\epsilon + \mu|} \\ &= \frac{1}{2\pi^2 a^2 t \gamma} \int_{-\infty}^{+\infty} \frac{dx}{x^2 + 1} \left( \ln \frac{16t}{\gamma} - \ln \left| x + \frac{\mu}{\gamma} \right| \right). \end{aligned} \quad (\text{A5})$$

Completing the integral over  $x$  in Eq. (A5), and noting that

$$\int_{-\infty}^{+\infty} dx \frac{\ln |x + \mu/\gamma|}{x^2 + 1} \approx \int_{-\infty}^{+\infty} dx \frac{\ln |x|}{x^2 + 1} = 0, \quad (\text{A6})$$

we immediately obtain Eq. (A1).

Similarly, a use of Eq. (2.5) leads to

$$\begin{aligned} \sum_{\mathbf{k}} (G_{\mathbf{k}}^R)^m (G_{\mathbf{k}}^A)^n &= \sum_{\mathbf{k}} \frac{1}{(-\epsilon_{\mathbf{k}} + i\gamma)^m (-\epsilon_{\mathbf{k}} - i\gamma)^n} \\ &\approx \frac{(-1)^{m+n}}{2\pi^2 a^2 t \gamma^{m+n-1}} \int_{-\infty}^{+\infty} dx \frac{\ln(16t/\gamma) - \ln |x|}{(x-i)^m (x+i)^n}. \end{aligned} \quad (\text{A7})$$

In the case of  $\ln(16t/\gamma) \gg 1$ , the second term in the right side of Eq. (A7) can be neglected. Thus, a completion of the integral over  $x$  in Eq. (A7) leads to Eq. (A2).

By means of Eqs. (2.2) and (2.5), we get

$$\sum_{\mathbf{k}} v_{\mathbf{k}}^2 (G_{\mathbf{k}}^R)^m (G_{\mathbf{k}}^A)^n = \int_{-4t}^{4t} \frac{d\epsilon}{(2\pi)^2} \frac{1}{(-\epsilon + i\gamma)^m (-\epsilon - i\gamma)^n} \int_{\epsilon_{\mathbf{k}}=\epsilon} \frac{ds}{v_{\mathbf{k}}} v_{\mathbf{k}}^2, \quad (\text{A8})$$

where the integration  $\int ds$  is over the constant-energy surface  $\epsilon_{\mathbf{k}} = \epsilon$ . Since the main contribution of the right side of Eq. (A8) comes from the nearly-nested surfaces of constant energy,

we have

$$\begin{aligned}\sum_{\mathbf{k}} v_{\mathbf{k}}^2 (G_{\mathbf{k}}^R)^m (G_{\mathbf{k}}^A)^n &\approx \frac{1}{2\pi} \frac{(m+n-2)!}{(m-1)!(n-1)!} \times \frac{i^{n-m}}{(2\gamma)^{m+n-1}} \times 4 \int_0^{\pi/a} \sqrt{2} dk_x 2\sqrt{2}ta \sin k_x a \\ &= \frac{16t}{\pi} \frac{(m+n-2)!}{(m-1)!(n-1)!} \times \frac{i^{n-m}}{(2\gamma)^{m+n-1}}.\end{aligned}\quad (\text{A9})$$

A substitution of Eq. (3.10) into Eq. (A9) immediately yields Eq. (A3).

## Appendix B: Vanishing QI correction to dc conductivity from the RR-correlation function

In this appendix, we will show that the RR-correlation function from the diagrams with  $\pi$ -mode cooperon or diffuson contributes no correction to the dc conductivity.

The contributions of the  $\pi$ -mode cooperon to the RR-correlation function come from Figs. 2(a), 2(c), and 2(d), which can be evaluated as

$$\begin{aligned}\Pi(\epsilon, \epsilon')_{2a}^{RR} &= e^2 \sum_{\mathbf{k}\mathbf{q}} \mathbf{v}_{\mathbf{k}} \cdot \mathbf{v}_{\mathbf{k}} (G_{\mathbf{k}}^R)^3 G_{\mathbf{Q}-\mathbf{k}}^R C_{\pi}(\mathbf{q}; \epsilon, \epsilon)^{RR} \\ &= -e^2 \sum_{\mathbf{k}\mathbf{q}} v_{\mathbf{k}}^2 (G_{\mathbf{k}}^R)^3 G_{\mathbf{k}}^A C_{\pi}(\mathbf{q}; \epsilon, \epsilon)^{RR},\end{aligned}\quad (\text{B1})$$

$$\begin{aligned}\Pi(\epsilon, \epsilon')_{2c}^{RR} &= e^2 \sum_{\mathbf{k}\mathbf{q}} \mathbf{v}_{\mathbf{k}} \cdot \mathbf{v}_{\mathbf{Q}-\mathbf{k}} (G_{\mathbf{k}}^R G_{\mathbf{Q}-\mathbf{k}}^R)^2 C_{\pi}(\mathbf{q}; \epsilon, \epsilon')^{RR} \\ &= e^2 \sum_{\mathbf{k}\mathbf{q}} v_{\mathbf{k}}^2 (G_{\mathbf{k}}^R G_{\mathbf{k}}^A)^2 C_{\pi}(\mathbf{q}; \epsilon, \epsilon')^{RR},\end{aligned}\quad (\text{B2})$$

and

$$\begin{aligned}\Pi(\epsilon, \epsilon')_{2d}^{RR} &= e^2 \sum_{\mathbf{k}\mathbf{k}'\mathbf{q}} \mathbf{v}_{\mathbf{k}} \cdot \mathbf{v}_{\mathbf{k}'} (G_{\mathbf{k}}^R G_{\mathbf{k}'}^R)^2 G_{\mathbf{Q}+\mathbf{q}-\mathbf{k}}^R G_{\mathbf{Q}+\mathbf{q}-\mathbf{k}'}^R C_{\pi}(\mathbf{q}; \epsilon, \epsilon)^{RR} C_{\pi}(\mathbf{q}; \epsilon, \epsilon')^{RR} \\ &= \frac{e^2}{4} \sum_{\mathbf{k}\mathbf{k}'\mathbf{q}} q^2 v_{\mathbf{k}}^2 v_{\mathbf{k}'}^2 (G_{\mathbf{k}}^R G_{\mathbf{k}}^A G_{\mathbf{k}'}^R G_{\mathbf{k}'}^A)^2 C_{\pi}(\mathbf{q}; \epsilon, \epsilon)^{RR} C_{\pi}(\mathbf{q}; \epsilon, \epsilon')^{RR}.\end{aligned}\quad (\text{B3})$$

A use of Eqs. (A2) and (A3) into Eqs. (B1)–(B3) immediately yields

$$\Pi(\epsilon, \epsilon')_{2a}^{RR} = - \sum_{\mathbf{q}} \frac{2e^2 D}{Dq^2 - i2(\epsilon + \mu) + 2\gamma_i}, \quad (\text{B4})$$

$$\Pi(\epsilon, \epsilon')_{2c}^{RR} = - \sum_{\mathbf{q}} \frac{4e^2 D}{Dq^2 - i(\epsilon + \epsilon' + 2\mu) + 2\gamma_i}, \quad (\text{B5})$$

and

$$\Pi(\epsilon, \epsilon')_{2d}^{RR} = \sum_{\mathbf{q}} \frac{4e^2 D^2 q^2}{[Dq^2 - i2(\epsilon + \mu) + 2\gamma_i][Dq^2 - i(\epsilon + \epsilon' + 2\mu) + 2\gamma_i]}. \quad (\text{B6})$$

The contributions of the  $\pi$ -mode diffuson to the RR-correlation function come from Figs. 3(a) and 3(c)–3(e), which can be similarly calculated as

$$\begin{aligned}\Pi(\epsilon, \epsilon')_{3a}^{RR} &= e^2 \sum_{\mathbf{k}q} \mathbf{v}_{\mathbf{k}} \cdot \mathbf{v}_{\mathbf{k}} (G_{\mathbf{k}}^R)^3 G_{\mathbf{Q}+\mathbf{k}}^R W^{RR} \Lambda^R(\mathbf{q}, \epsilon)^2 \\ &= -e^2 \sum_{\mathbf{k}q} v_{\mathbf{k}}^2 (G_{\mathbf{k}}^R)^3 G_{\mathbf{k}}^A W^{RR} \Lambda^R(\mathbf{q}, \epsilon)^2,\end{aligned}\quad (\text{B7})$$

$$\begin{aligned}\Pi(\epsilon, \epsilon')_{3c}^{RR} &= e^2 \sum_{\mathbf{k}q} \mathbf{v}_{\mathbf{k}} \cdot \mathbf{v}_{\mathbf{Q}+\mathbf{k}} (G_{\mathbf{k}}^R G_{\mathbf{Q}+\mathbf{k}}^R)^2 W^{RR} \Lambda^R(\mathbf{q}, \epsilon) \Lambda^R(\mathbf{q}, \epsilon') \\ &= -e^2 \sum_{\mathbf{k}q} v_{\mathbf{k}}^2 (G_{\mathbf{k}}^R G_{\mathbf{k}}^A)^2 W^{RR} \Lambda^R(\mathbf{q}, \epsilon) \Lambda^R(\mathbf{q}, \epsilon'),\end{aligned}\quad (\text{B8})$$

$$\begin{aligned}\Pi(\epsilon, \epsilon')_{3d}^{RR} &= e^2 \sum_{\mathbf{k}\mathbf{k}'q} \mathbf{v}_{\mathbf{k}} \cdot \mathbf{v}_{\mathbf{k}'} (G_{\mathbf{k}}^R G_{\mathbf{k}'}^R)^2 G_{\mathbf{Q}+\mathbf{q}+\mathbf{k}}^R G_{\mathbf{Q}+\mathbf{q}+\mathbf{k}'}^R W^{RR} \Lambda^R(\mathbf{q}, \epsilon)^2 D_{\pi}(\mathbf{q}; \epsilon, \epsilon')^{RR} \\ &= \frac{e^2}{4} \sum_{\mathbf{k}\mathbf{k}'q} q^2 v_{\mathbf{k}}^2 v_{\mathbf{k}'}^2 (G_{\mathbf{k}}^R G_{\mathbf{k}}^A G_{\mathbf{k}'}^R G_{\mathbf{k}'}^A)^2 W^{RR} \Lambda^R(\mathbf{q}, \epsilon)^2 D_{\pi}(\mathbf{q}; \epsilon, \epsilon')^{RR},\end{aligned}\quad (\text{B9})$$

and

$$\begin{aligned}\Pi(\epsilon, \epsilon')_{3e}^{RR} &= e^2 \sum_{\mathbf{k}\mathbf{k}'q} \mathbf{v}_{\mathbf{k}} \cdot \mathbf{v}_{\mathbf{k}'} (G_{\mathbf{k}}^R G_{\mathbf{k}'}^R)^2 G_{\mathbf{Q}+\mathbf{q}+\mathbf{k}}^R G_{-\mathbf{Q}-\mathbf{q}+\mathbf{k}'}^R W^{RR} \Lambda^R(\mathbf{q}, \epsilon) \Lambda^R(\mathbf{q}, \epsilon') D_{\pi}(\mathbf{q}; \epsilon, \epsilon')^{RR} \\ &= -\frac{e^2}{4} \sum_{\mathbf{k}\mathbf{k}'q} q^2 v_{\mathbf{k}}^2 v_{\mathbf{k}'}^2 (G_{\mathbf{k}}^R G_{\mathbf{k}}^A G_{\mathbf{k}'}^R G_{\mathbf{k}'}^A)^2 W^{RR} \Lambda^R(\mathbf{q}, \epsilon) \Lambda^R(\mathbf{q}, \epsilon') D_{\pi}(\mathbf{q}; \epsilon, \epsilon')^{RR},\end{aligned}\quad (\text{B10})$$

leading to

$$\Pi(\epsilon, \epsilon')_{3a}^{RR} = - \sum_{\mathbf{q}} \frac{4e^2 D \gamma}{[Dq^2 - i2(\epsilon + \mu) + 2\gamma_i]^2}, \quad (\text{B11})$$

$$\Pi(\epsilon, \epsilon')_{3c}^{RR} = \sum_{\mathbf{q}} \frac{8e^2 D \gamma}{[Dq^2 - i2(\epsilon + \mu) + 2\gamma_i][Dq^2 - i2(\epsilon' + \mu) + 2\gamma_i]}, \quad (\text{B12})$$

$$\Pi(\epsilon, \epsilon')_{3d}^{RR} = \sum_{\mathbf{q}} \frac{8e^2 D^2 q^2 \gamma}{[Dq^2 - i2(\epsilon + \mu) + 2\gamma_i]^2 [Dq^2 - i(\epsilon + \epsilon' + 2\mu) + 2\gamma_i]}, \quad (\text{B13})$$

and

$$\Pi(\epsilon, \epsilon')_{3e}^{RR} = - \sum_{\mathbf{q}} \frac{8e^2 D^2 q^2 \gamma}{[Dq^2 - i2(\epsilon + \mu) + 2\gamma_i][Dq^2 - i2(\epsilon' + \mu) + 2\gamma_i][Dq^2 - i(\epsilon + \epsilon' + 2\mu) + 2\gamma_i]}. \quad (\text{B14})$$

Substituting Eqs. (B4)–(B6) into Eqs. (4.3) and (4.34), one can easily show that

$$2\sigma(T)_{2a}^{RR} = \sigma(T)_{2c}^{RR} = -\sigma(T)_{2d}^{RR} = \frac{e^2}{8\pi^2} \int_{-\infty}^{+\infty} \frac{dx}{\cosh^2 x} \ln \frac{\gamma^2}{(2Tx + \mu)^2 + \gamma_i^2} \quad (\text{B15})$$

and

$$2\sigma(L)_{2a}^{RR} = \sigma(L)_{2c}^{RR} = -\sigma(L)_{2d}^{RR} = \frac{e^2}{4\pi^2} \ln \frac{\gamma^2}{\gamma_L^2 + \mu^2}, \quad (\text{B16})$$

leading to

$$\sigma_{\pi\text{-coop}}^{RR} = 2\sigma_{2a}^{RR} + \sigma_{2c}^{RR} + 2\sigma_{2d}^{RR} = 0. \quad (\text{B17})$$

From Eqs. (4.3) and (B11)–(B14), it follows that

$$2\sigma_{3a}^{RR} = -\sigma_{3c}^{RR} \text{ and } \sigma_{3d}^{RR} = -\sigma_{3e}^{RR}, \quad (\text{B18})$$

yielding

$$\sigma_{\pi\text{-diff}}^{RR} = 2\sigma_{3a}^{RR} + \sigma_{3c}^{RR} + 2\sigma_{2d}^{RR} + 2\sigma_{2d}^{RR} = 0. \quad (\text{B19})$$

Equations (B17) and (B19) indicate that the RR-correlation function from the contribution of the  $\pi$ -mode cooperon or diffuson has a vanishing correction to the dc conductivity.

## References

- [1] P. W. Anderson, Phys. Rev. **109**, 1492 (1958).
- [2] E. Abrahams, P. W. Anderson, D. C. Licciardello, and T. V. Ramakrishnan, Phys. Rev. Lett. **42**, 673 (1979).
- [3] N. F. Mott and W. D. Twose, Adv. Phys. **10**, 107 (1961).
- [4] V. L. Berezinskii, Sov. Phys. JETP **38**, 620 (1974).
- [5] P. A. Lee and T. V. Ramakrishnan, Rev. Mod. Phys. **57**, 287 (1985).
- [6] A. Puri and T. Odagaki, Phys. Rev. B **24**, 5541 (1981).
- [7] A. Fertis, A. Andriotis, and E. N. Economou, Phys. Rev. B **24**, 5806 (1981).
- [8] C. M. Soukoulis, I. Webman, G. S. Grest, and E. N. Economou, Phys. Rev. B **26**, 1838 (1982).
- [9] A. Eilmes, R. A. Römer, and M. Schreiber, Eur. Phys. J. B **1**, 29 (1998).
- [10] S. J. Xiong and S. N. Evangelou, Phys. Rev. B **64**, 113107 (2001).
- [11] H. Potempa and L. Schweitzer, Phys. Rev. B **65**, 201105(R) (2002).
- [12] G. Bergmann, Phys. Rep. **107**, 1 (1984).
- [13] B. Kramer and A. MacKinnon, Rep. Prog. Phys. **56**, 1969 (1993).
- [14] D. V. Khveshchenko, A. G. Yashenkin, and I. V. Gornyi, Phys. Rev. Lett. **86**, 4668 (2001).
- [15] A. G. Yashenkin, W. A. Atkinson, I. V. Gornyi, P. J. Hirschfeld, and D. V. Khveshchenko, Phys. Rev. Lett. **86**, 5982 (2001).

- [16] P. J. H. Denteneer, R. T. Scalettar, and N. Trivedi, Phys. Rev. Lett. **87**, 146401 (2001).
- [17] P. A. Lee, Phys. Rev. Lett. **71**, 1887 (1993).
- [18] Y. H. Yang, D. Y. Xing, M. Liu, and Y. G. Wang, Phys. Rev. B **66**, 205310 (2002).
- [19] G. D. Mahan, *Many-Particle Physics* (Plenum Press, New York, 1981), p. 243.
- [20] B. L. Altshuler, A. G. Aronov, and P. A. Lee, Phys. Rev. Lett. **44**, 1288 (1980).
- [21] H. Fukuyama, J. Phys. Soc. Japan **50**, 3407 (1981).
- [22] B. L. Altshuler, A. G. Aronov, A. I. Larkin, and D. E. Khmel'nitskii, Sov. Phys. JETP **54**, 411 (1981).